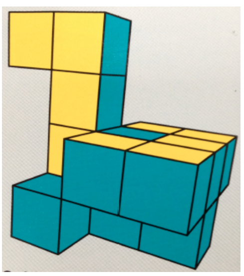
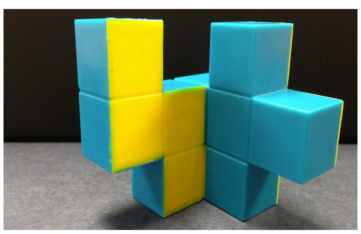
Math for Young Children (M4YC)
WCDSB Lesson Study Team
St. Bernadette Catholic School

Public Research Lesson:
May 17, 2013



"Exploring 3D Pentominoes"



Research Questions:

How do students move from 2D thinking to 3D thinking (rotations and orientation and location)?
How do students interpret photos of 3D figures compared to diagrams of 3D figures?
How can we prompt students to use more gestures in their mathematics?

Teacher-Researcher Team:

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TRENT UNIVERSITY

Ontario

BACKGROUND

How did we get started?

We were asked if we wanted to participate in some math research. We are all teachers from the same school who have worked to build a team approach where we share ideas with a common goal of improving our students' understanding of mathematics. Our team decided this was a meaningful professional development opportunity to improve our teaching practice and understanding of how children learn math. This journey began in January where we set goals and began our explorations together.

Why did we choose this content area?

As a beginning exercise we each reflected on which strands of math we felt we spent the most time on and had the most PD on. We found out that every one of us selected Number and Patterns as the two strands we spend the most time on, and Geometry was at or near the bottom of the list. This made us think that Geometry might be a good focus. We read the literature review on Math for Young Children and this got us excited about what young children can do, and helped us reconsider the role of play and exploration in math. We learned that the consensus in the literature is that "play does not guarantee mathematical development but it offers rich possibilities" (NAYCM/NCTM, 2002). Students who come from underprivileged circumstances come to school with limited experiences, and lag behind in mathematics, partly because of a lack of math-talk prior to coming to school (Levine et. al. 2010). We wanted to have a deeper understanding of how to present Geometry concepts in playful and engaging ways. We also wanted to link current research on spatial reasoning with our practice. We determined that spatial reasoning is a pillar of mathematics and we know that it is a predictor of later success in math. We want our students to be empowered as capable little mathematicians, and to think about their thinking (metacognition).

What do we know from the current research?

- Spatial thinking generally involves the location of objects and our ability to manipulate them in different ways. It also includes our capacity to relate to and navigate the wider world around us. (Newcombe, 2010)
- "In our view, this spatial sense consists of three main components that are most essential for enabling young children to 'grasp the world' and to develop mathematical thinking: *spatial visualization*, geometry ('shapes' in short), and *spatial orientation* ('space' in short). These components can be recognized in the foundations of comprehensive mathematics curricula..." (VanNess, 2007)

Aspects of spatial reasoning that have been identified (from IOSTEM working group on Spatial Reasoning, 2013)

- Symmetry
- Balancing
- Locating
- Orienting
- Decomposing/recomposing

- Shifting dimension
- Diagramming
- Continuity/connectedness
- Navigating
- Transformations
- Comparing
- Scaling
- "Feeling"
- Visualizing

According to the Ontario Curriculum, spatial reasoning is important, but as teachers we tend to typically focus on the standard language of shape and attributes of shape:

- Spatial sense is the intuitive awareness of one's surroundings and the objects in them.
- Spatial sense is necessary for understanding and appreciating the many geometric aspects of our world.
- Students develop their spatial sense by visualizing, drawing, and comparing shapes and figures in various positions.

We know spatial reasoning is important because it is an indicator of success not only in mathematics, but also in general intelligence, language acquisition, the ability to solve non-routine problems, and capacity for the STEM careers. (Newcombe, 2010; Casey, Nutall & Pezaris, 1992, 1997, 2001; Wheatley et al. 1994; Casey and Erkut 2005, Clements & Sarama, 2011)

We were encouraged to learn that spatial reasoning is malleable and even adults can develop and improve their spatial reasoning. Spatial reasoning can be learned through effective programming starting in the early years, but also later (Feng, 2007; Uttal et al, 2012). As part of our learning, we have been exploring math activities as a group and analyzing the benefits of a variety of math manipulatives to improve our understanding of early years spatial reasoning.

Evolution of our Research Questions

Month 1:
What are the connections between geometry and spatial sense in their (students') world?
Do students know the language (e.g., attributes)?
Are the students able to compose and decomposed irregular shapes?

Month 2:
Does physical practice improve mental ability to compose/decompose and perform mental rotations? (Levine)
How does gesture help?
Do gestures fall off as students get older?

Month 3:
What is student understanding of transformations (rotations and reflections in particular)?
Are students doing enough composing and decomposing of shape? (Inverse Levine)
What are our own gestures and how are they affecting student thinking and student gestures?

Month 4:
How do students move from 2D thinking to 3D thinking (rotations and orientation and location)? (M4YC blocks transformation task)
How do students interpret photos of 3D figures compared to diagrams of 3D figures?
How can we prompt students to use more gestures in their mathematics?

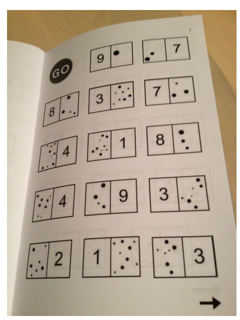
Key References:
Newcombe, Picture This
M4YC Lit review

CLINICAL INTERVIEWS

Observing children in clinical interviews and developing exploratory lessons has helped our team to inform our teaching and to learn more about how young children are thinking in mathematics, as well as the assets they bring to their learning. Because we implemented the exploratory lessons across multiple grades, we have been able to identify continua of abilities and ways of thinking and to more precisely anticipate what students might do.

Clinical interviews with our students have involved four different tasks:

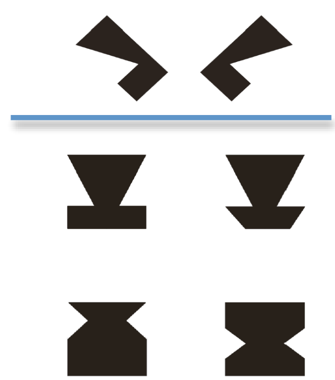
1) a magnitude test comparing symbolic and non-symbolic numbers and arrays (developed by Daniel Ansari);



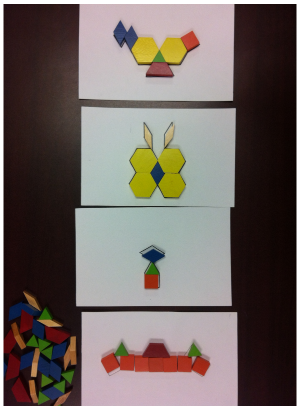
"Choose which is larger or means 'the most'…"

2) a 2D mental rotation task (developed by Susan Levine);

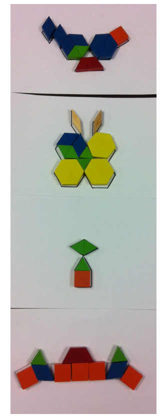
"If you put the two top shapes together, which picture will they make?"



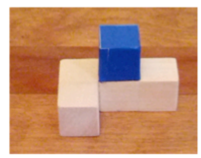
3) a fill-in task focusing on composing & decomposing shapes and scaling (teacher-generated), and;



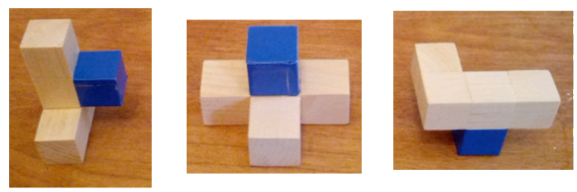
Final phase in these four items was not to scale to encourage analysis / composition of the shape that did not entirely rely on the outline.



4) a 3D mental rotation task (developed by Zack Hawes).

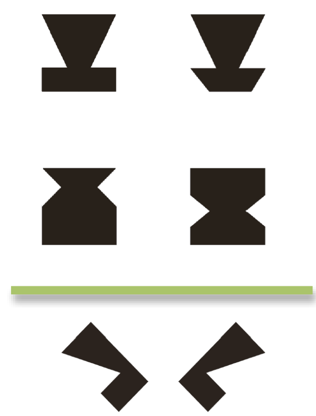


Which of the figures below can be rotated to match this figure.



The clinical interviews have helped us isolate some areas of focus within spatial reasoning and have influenced the development of our exploratory lessons.

For example, our observations and video analysis of students performing the 2D mental rotation "Levine" task made us curious about whether or not they would have more facility moving from whole to part than from part to whole. We designed a task we called the "Inverse Levine" to explore this further…



Observing students perform the 3D mental rotation task also had an impact on our thinking about early geometry and the kinds of experiences/reasoning that might benefit children.

EXPLORATORY LESSONS

Background: our exploratory lessons focused on several aspects of spatial reasoning and geometry. The following tasks focused on:

Free Play with Manipulatives
- Open-ended exploration of materials (e.g., tangrams, pattern blocks, cubes) was a springboard to developing exploratory lessons and clinical interviews

Tangrams/Pattern blocks
- Language/gesture of transformations
- Composing/decomposing shapes
- Scaling (fill-in tasks)
- Visualizing

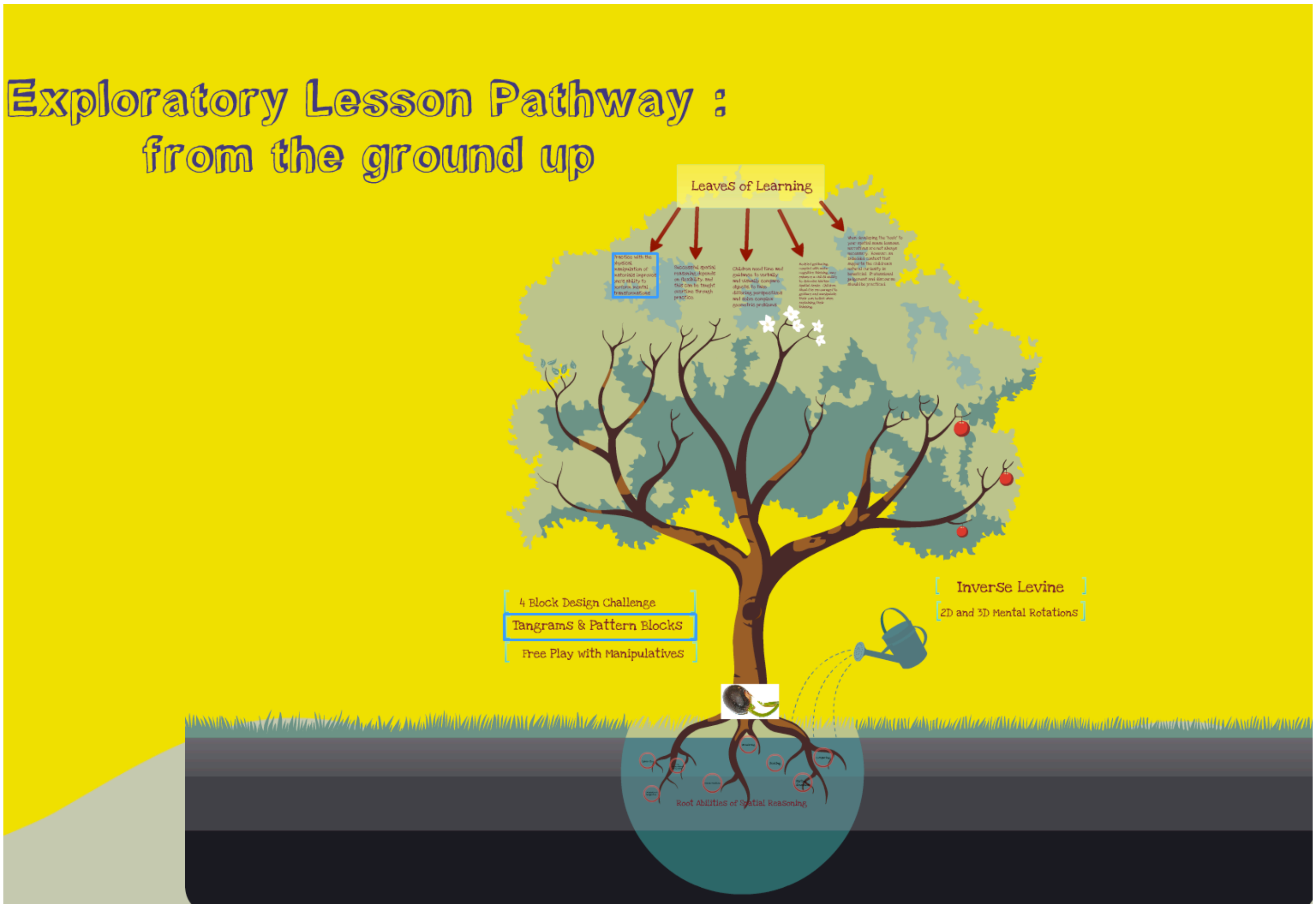
4-cube challenge
- Orientation/perspective taking
- Congruent vs. similar
- Comparing

2D & 3D mental rotations
- Orienting
- Visualizing
- Comparing
- Transformations
- Composing/recomposing
- Perspective taking

Inverse Levine
- Symmetry
- Composing and decomposing
- Orientation/transformation

Our learning from observations of students:
- Provided a framework for other exploratory lessons and the public lesson, acted as a spring board for play experiences;
- Demonstrated that prior experience to spatial reasoning tasks improved student perseverance and ability to compose and decompose shapes (which led to our interest in student ability to mentally rotate with pattern blocks/tangrams;
- Provided opportunities for rich talk to describe comparisons of one figure to another (4-cube challenge);
- Demonstrated that in some cases narrative can be distracting from the mathematics and may take away from non-verbal spatial reasoning (2D & 3D mental rotations), and led us to inquire about the context of playing with figures without a narrative as you will see in today's lesson;
- Highlighted the importance of gesture use (teacher and student) to support students' mental rotation abilities (Inverse Levine).

When you have a minute, we invite you to explore the display of tasks, photos, student work, etc. set up in the library.



PUBLIC LESSON

Main activity	Students are using 2D instructions (visual – diagram or picture) to build a 3D puzzle.
Students	• SK/1 combination (8 students) – 2 pairs from Grade 1, 2 pairs from SK • Boy/girl pairings, ability-matched • Mixed perceived math ability
Overall objective/goals	• To bridge students' experiences between the 2D and 3D work we've been doing • To build students' spatial reasoning skills in navigating between 2D and 3D worlds • To foster flexible thinking (composing & decomposing, moving between part-whole and whole-part investigation of shapes/figures) • To foster discussion and physical manipulations related to spatial thinking & transformational geometry: turns, flips, slides, rotations
Research questions	How do students move from 2D thinking to 3D thinking (rotations and orientation and location)? How do students interpret photos of 3D figures compared to diagrams of 3D figures? Does physical practice improve mental ability to compose/decompose and perform mental and physical rotations? Does having kinesthetic experience first help children visualize and then create? How can we prompt students to use more gestures in their mathematics? How does providing students with playful challenges foster their geometric thinking and language?
Criteria for task	• Must be a rich problem/challenge that requires them to think mathematically/spatially • Needs to be playful and rich enough to lead to extension for children who finish quickly • 3D focus including rotation and flipping
Materials	• 3D Pentomino Puzzle • photos • BrightLinks • trays
Observation guides	Student use of math language Student use of gesture Student approaches to task Invented contexts with use of manipulatives
KEY QUESTIONS	Does your structure look like the picture/photo? How do you know? What helped you?

PHASE I:

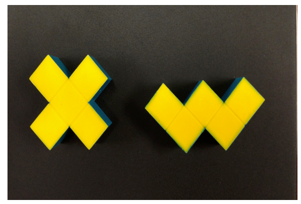
5 MINUTES OF EXPLORATORY PLAY in pairs – 5 minutes

Begin by pulling the figures out of a brown paper bag"
"I have a special set of blocks here. Remember the 5-cube challenge? These special blocks are similar except you can't take these apart."

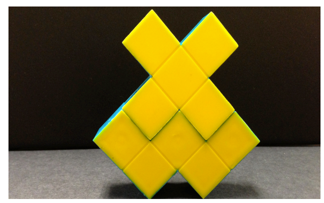
Key questions: what do you see, what do you notice?

PHASE II:

Challenge #1:
"You need these two blocks – find these two blocks in your set."



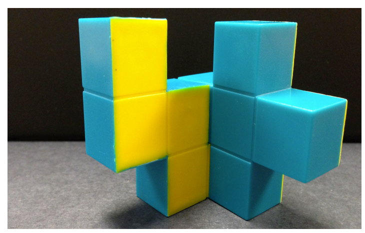
Show photo of "fish" and ask them to make it with their partner (but don't say "fish).



Key prompt: "Tell me what you did to make it…"

Anticipated responses: positional language, location, labeling/naming of blocks, overall structure, edges, corners, colour, gestalt (connections to schema)

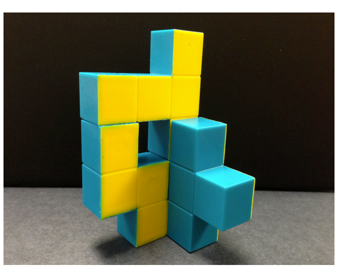
Challenge #2:



"Here are the same two pieces…but now they look different!
Transform these blocks to look like this.
Before you make it with your partner, don't touch the pieces:
Turn and talk to your partner.
What will you have to do to make it look like the new figure?"
[Observers watch for gestures and language.]

Key prompt: "What will you have to do to make it look like the photo?

Challenge #3:



Show picture of 3rd structure (same as challenge 2 but with 1 more piece added in).

"Pick one of these two blocks from your set and make it with your partner."

[Show picture with 2 separate blocks; have students choose which is the additional piece.]

Key prompt: "Predict which of the two pieces you think it is!"

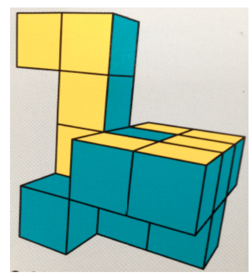
Challenge #4:
Show the tower structure – "find the pieces out of full set of 12 and build it with your partner."
(It is made from 4 pieces – no blocks preselected.)

Scaffolding: give the 4 pieces if students are really struggling

PHASE III:

Challenge #5:
"Now make one with the same 4 pieces that's different." (2 minutes)
Teaching team quickly posts photos on whiteboard (while photos are being posted, students can build a different one with the same 4 pieces).

"Pick one that's not your own and recreate."



Challenge #6 (if time/students not "tapped out"):
"Look at this drawing. Use 3 pieces to make this structure."
(Number 2.11 in booklet.)

Give copy of booklet to early finishers and encourage them to build one puzzle of their selection.

DISCUSSION QUESTIONS (if time, feels appropriate)
How did you know which pieces to pick?
What did you like?
What was fun?
What was hard?
What are you most proud of?
Was it easier to look at the photo or the diagram?
If a friend were playing, what hint/tip would you give?
If you were to tell mom and dad what you did today, what would you tell them?

Observation Guide 1:

Student Use of Math Language

Consider making a word count for each of the words below and other math vocabulary for the students you are observing.

Examples	Tally	Key words or phrases
Transformation words (flip, rotate, slide, turn, etc)		
Location words (on top, beside, behind)		
Orientation or arrangement words (from the top, side, back, front)		
Symmetry words (same on both sides, same as the other half or other side)		
Composing/decomposing/recomposing words (take apart, put together, put back together, this part goes into this part)		
Property words (edges, corners, colour, faces)		
Shape words (cubes, squares, rectangles, prisms)		

Other math language you hear:

Observation Guide 2:

Evidence of Gestures and Movement: Consider focusing on *either* gestures or movement

Types of Gestures/Movement	Situation (when)	Any accompanying language?
Hand gestures: Pointing		
Hand gestures: Moving and turning		
Hand gestures: Whole object - Big motions		
Movement: Change in body position (getting up move around)		
Movement: of tray (students move the tray)		
Movement of head: tilting of head, orienting line of vision		
Movement of Materials: Matching (physical overlay) materials with images provided		

Observation Guide 3:

Manipulatives and invented contexts

A) What kinds of contexts or prior experiences did the students overlay on the manipulatives? (i.e., “hey this block looks like an ‘x’ and this looks like a ‘w’”, “you have to make a spaceship”)	B) Were student invented contexts distracting or enabling to their math thinking?

Observation Guide 4:

How are students approaching the task? Consider focusing on one of these four questions.

Strategies: What strategies are students using? (trial and error, visualizing then trying, whole object rotations or individual (parts) figure rotations)

Persistence: Does it wax or wane depending on the difficulty of the tasks or time they have spent on the task?

Attempts: As tasks become more difficult do students increase or decrease the number of attempts?

Flexibility: Can students think flexibly between part-to-whole or from whole-to-part?